2021

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×12

   a) What is the Taylor's series expansion of sin x ?

   b) What is the value of $\lim_{x \to \infty} x^{1/x}$?

   c) Define Cauchy's mean value Theorem.

   d) What is the radius of convergence of $\sum x^n$?

   e) Define Bernstelin polynomial.

   f) What is the interval of convergence of $\sum \frac{(x-1)^n}{2^n}$?

   g) If f(x) = C. What is the value of $\int_a^b f(x)dx$?

h) Define lower Darboux integral.

i) What is the value of U(p, f) – L(p, f) ?

j) What is the value of $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$?

k) In which interval the series $\sum 2^n (\alpha)^{2n-1}$ converges uniformity ?

l) Define absolute convergence.

Part-II

2. Answer any *eight* of the following : 2 × 8

a) Examine the convergence of $\int_0^1 \frac{dx}{x^2}$.

b) Find the value of m and n such that $\int_0^1 e^{-mx} x^n dx$ converge ?

c) Using s(f, p, t) calculate $\int_a^b \frac{dx}{x^2}$, $a > 0$.

d) Show that

$$\int_1^{\alpha} \frac{dt}{t} = \log \alpha.$$

e) Show that the series $\sum_{k=1}^{\infty} \left(xe^{-x}\right)^k$ is uniformly convergent in [0, 2].

f) Show that the sum function $s(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{\alpha}}$ is uniformly continuous of [–1, 1] for $\alpha > 1$.

g) Show that

$$\lim_{x \to 0} x^x = 1.$$

h) Find the local maximum and minimum of $f(x) = 8x^5 - 15x^4 + 10x^2$.

i) Verify Lagrange mean value condition for $f(x) = x^2 + 2x + 3$ on [4, 6].

j) Show that $f_n(x) = \frac{nx + x^2}{n^2}$ converges pointwise.

Part-III

3. Answer any ***eight*** of the following : 3×8

a) Show that $1 - \frac{1}{2}x^2 \le \cos x$.

b) Use Taylor's theorem with $n = 2$ to approximate $\sqrt[3]{1+x},\ x > -1$.

c) Show that $f(x) = \sin x,\ x \in \left[0, \frac{\pi}{4}\right]$ is approximated by a polynomial $p(x) = x - \frac{x^3}{6}$ with an error less than $\frac{1}{400}$.

d) Show that for $-1 < x \le 1$

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n-1}\frac{x^n}{n}$$

in particular $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$

e) Show that for every

$$x \in R, \sum_{k=0}^{n}(nx - k)^2\binom{n}{k}x^k(1 - x)^{n-k}$$

$$= nx(1 - x) \le \frac{n}{4}.$$

f) If $h(x) = x$ for $x \in [0, 1]$ show that $h \in R[0, 1]$.

g) Show that $\int_0^t \sin x \, dx = 1 - \cos t$.

h) Show that $\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$ exists if and only if $n < m + 1$.

i) Examine the converage of $\int_0^{\infty} \left(\frac{1}{x} - \frac{1}{\sinh x} \right) \frac{dx}{x}$

j) If f is integrable on [a, b] show that f^2 is also integrable on [a, b].

Part-IV

4. a) Define Maclaurin's series expansion. Show that for $|x| < 1, \alpha \in R$

$$(1+x)^{\alpha} = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \ldots$$

where

$$\binom{\alpha}{0} = 1, \binom{\alpha}{k} = \frac{\alpha(\alpha-1)\ldots.(\alpha-k+1)}{k!}, k \in N.$$ 7

OR

b) Let $f : I \to R$ be twice continuously differentiable on I. The f'' is continuous on I. Let $f'(c) = 0$ for some $c \in I$. Then show that f has

i) Local maximum at C if $f''(c) < 0$.

ii) Local minimum at C if $f''(c) > 0$. Prove that the largest rectangle inscribed in a circle is a square.

5. a) Let $f \in B\,[a, b]$ be continuous over [a, b] except over a finite set. Then show that f is also integrable over [a, b]. 7

OR

b) Let $f \in R[a, b]$ then show that $|f| \in R[a, b]$.

Again $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx$.

6. a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ exist if and only if m, n are both positive. 7

OR

b) Discuss the convergence of $\int_0^1 \log \sqrt{x}\, dx$ and hence evaluate it.

7. a) State the prove Abel's limit theorem. 7

OR

b) State and prove Weierstrass approximation theorem.

2021

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×12

   a) What are the generators of z under ordinary addition ?

   b) What is the order of A_n for $n > 1$?

   c) If $n = 6$, what is the value of $\phi(n)$?

   d) What is the identity element of Z_n under addition Mod n ?

   e) What is the order of a group of integers under addition ?

   f) In Z, what is the value of $\langle 1 \rangle$?

   g) How many elements of order 5 are in $Z_{25} \oplus Z_5$?

   h) Write down the value of $|(g_1, g_2, \cdots g_n)|$

   i) What is the value of k, if $20^6 \equiv k \pmod 7$?

   j) What is the Kernel of $\phi : Z \rightarrow Z_n$?

k) In group Q of all quaterions which quaternior form the center of Q ?

l) Define $C(R_{90})$ in D_4.

Part-II

2. Answer any ***eight*** of the following : 2×8

a) Explain why the set of integers under multiplication is not a group ?

b) What is the Caley table for U(10) ?

c) Show that for group elements a and b $(ab)^{-1} = b^{-1}a^{-1}$.

d) Show that 3 is a generator of Z_8.

e) Decompose (1, 3, 2, 4) . (1, 7, 6, 2) into transposition.

f) Let $G = Z_9$ and $H = \{0, 3, 6\}$. List down the left cosets of H in G ?

g) List down the elements of $U(8) \oplus U(10)$.

h) Find whether $\phi(x) = x^2$ from R(t) to itself is a homomorphism or not ?

i) If $|H| = n$, Show that $|\phi(H)|$ devides ?

j) If $h(g_1) = h(g_2)$ prove that $g_1 = g_2$.

Part-III

3. Answer any ***eight*** of the following : 3×8

a) Show that $(Z/N) \approx Z_N$.

b) For $\phi : Z_{12} \rightarrow Z_{12}$ verify that $\phi(x) = 3x$ is a homomorphism. Find ker (ϕ) ?

c) Let $\phi : G \rightarrow G$ is a homomorphism and $g \in G$. Prove that if $\phi(g) = g'$ then $\phi^{-1}(g^1) = \{x \in G | \phi(x) = g^1\} = g \ker f$.

d) Using a example show that the converse of Lagrange's theorem is false.

e) Show that A_4 has no subgroup of order 6.

f) Prove that a group of order 5 must be cyclic.

g) Show that the symmetric group S_3 under composition is non abelian.

h) Prove that for $n \geq 3$ the subgroup generated by 3-cycle is A_n.

i) Construct a Caley table for D_4.

j) if G ia an abelian group under multiplication show that $H = \{x^2 | x \in G\}$ is subgroup of G.

Part-IV

4\. a) With pictures and words describe each symmetry in D_3. Show that D_3 is a non-abelian group. 7

OR

b) Prove that for each $a \in G$, $C(a)$ is a subgroup of G.

5\. a) Prove that in a finite group the number of elements of order d is divisible by $\phi(d)$. Find $\phi(40)$ and $\phi(75)$. 7

OR

b) Prove that every permutation can be written as a cycle or as a product of disjoint cycles.

6\. a) State and prove Lagrange's theorem. 7

OR

b) Let G and H be finite cyclic group. Then prove that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

7\. a) State and prove N/C theorem. Show that every group of order 77 is cyclic. 7

OR

b) State and prove first isomorphism theorem. Find all homomorphism from Z_{12} to Z_{30}.

III-UG-Math-(CC)-VII (NC)

2021

Full Marks - 60

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×8

   a) What is the characteristic equation of f(x, y, u) in nonparametric form ?

   b) What is the tangent vector to the parametric curve x(t), y(t), u(t) ?

   c) Write down the one dimensional wave equation.

   d) What is the canonical form of a hyperbolic equation.

   e) For positive $k = p^2$ what is the general solution of $x'' - kx = 0$?

   f) What is the eigen function of the vibrating string ?

g) Give an example of a normal differential equation in Z.

h) For $x = e^{2t}$, what is the value of $(D - 2)^2 x$?

Part-II

2. Answer any ***eight*** of the following : $1\frac{1}{2} \times 8$

a) Write down the general form of semilinear partial differential equation.

b) Define characteristic base curve.

c) Eliminate the arbitrary function from

$z = x + y + f(xy)$

d) Determine the region in which $xu_{xx} + u_{yy} = x^2$ is elliptic.

e) Classify the curve $u_{xx} + yu_{yy} - xu_y + y = 0$.

f) Define domain of influence.

g) When the initial displacement of a wave is zero what is the d' Alembert Solution.

h) Verify the linear independence of

$x = e^{5t}$ $\quad x = e^{3t}$

and

$y = -3e^{5t}$ $\quad y = -e^{3t}$

i) What is the general solution of

$x = f_1(t)$ $\quad x = f_2(t)$

$y = g_1(t)$ $\quad y = g_2(t)$

j) Find the characteristics of $u_{xy} + u_x + u_y = 3x$

Part-III

3. Answer any ***eight*** of the following : 2 × 8

a) Solve the initial value problem

$u_t + uu_x = x$

$u(x, 0) = f(x)$ where $f(x) = 1$

b) Show that the family of spheres $x^2 + y^2 + (z-c)^2 = r^2$ satisfies the first order linear differential equation $yp - xq = 0$

c) Find the solution of $u(x + y)\, u_x + u(x - y)u_y = x^2 + y^2$ with cauchy data $u = 0$ on $y = 2x$.

d) Fine the cononical form of

$$u_{xx} + (2 \operatorname{cosec} y)u_{xy} + (\operatorname{cosec}^2 y)u_{yy} = 0$$

e) Determine the general solution of

$$u_{xx} + 10u_{xy} + 9 u_{yy} = y$$

f) Determine the solution of the initial value problem $u_{tt} - C^2 u_{xx} = 0$ $u(x, 0) = x^3$, $u_t(x, 0) = x$

g) Determine the solution of

$$u_{tt} = 4u_{xx}, \quad x > 0, \quad t > 0$$

$$u(x, 0) = |\sin x|, \quad x > 0$$

$$u_t(x, 0) = 0, \quad x \geq 0$$

$$u(x, 0) = 0, \quad t \geq 0 \text{ for } x > 2t.$$

h) What is the solution of

$$x = e^{5t} \qquad\qquad x = e^{3t}$$

and

$$y = -3e^{5t} \qquad\qquad y = -e^{3t}$$

i) Solve $(D^2 - 2D + 1)\, y = e^{-x}$

j) Solve $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 4y = 0$

Part-IV

4\. a) Show that the general solution of the linear equation 6

$(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$ is

$u(x, y, z) = f(x+y+z, x^2+y^2+z^2)$

OR

b) Find the integral surface of $uu_x + u_y = 1$ So that the curve passes through an initial curve represented parametrically $x = x_0(s)$, $y = y_0(s)$, $u = u_0(s)$ where S is a parameter.

5\. a) Consider the wave equation $u_{tt} - c^2 u_{xx} = 0$ where c is a constant. Find the general solution. 6

OR

b) Use $u = f(\xi)$, $\xi = \dfrac{x}{\sqrt{4kt}}$ to solve the parabolic system $u_t = ku_{xx}$ $-\infty < x < \infty$, $t > 0$.

$u(x, 0) = 0, \quad x < 0$

$u(x, 0) = u_0, \quad x > 0$

6. a) Find out d'Alembert solution of

$$u_{tt} = C^2 u_{xx}, \quad 0 < x < \infty, \quad t > 0$$

$$u(x, 0) = f(x), \quad 0 \le x < \infty$$

$$u_t(x, 0) = g(x), \quad 0 \le x < \infty$$

$$u_x(0, t) = 0, \quad 0 \le t < \infty$$ 6

OR

b) Solve the Cauchy problem for the non-homogeneous equation

$$u_{tt} = C^2 u_{xx} + h^*(x, t)$$

with initial condition

$$u(x, 0) = f(x), \quad u_t(x, 0) = g^*(x)$$

7. a) Use operator method of solve

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$$

and $$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$$ 6

OR

b) Solve $\frac{dx}{dt} = 6x - 3y$

$$\frac{dx}{dt} = 2x + y$$

III-UG-Math-(GE-B)-I (NC)

2021

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×12

   a) What is the radius of curvature of a polar curve $r = f(\theta)$?

   b) Define enveloping cylinder.

   c) Define great circle.

   d) What is the Maclaurin series expansion of $\log\ (1 + x)$?

   e) Give an example of monotonically decreasing function.

   f) Define Rolle's theorem.

g) What is the domain of $f(x, y) = \sqrt{\dfrac{x+y}{x-y}}$.

h) State the necessary condition for differentiability of a function f(x,y) at (a, b).

i) Write the condition for which $f_{yx} = f_{xy}$ for the function f(x, y).

j) What is the order and degree of $x^2y''' - \sqrt{xy} = 0$.

k) What is the value of $\dfrac{1}{D^2 + a^2}\sin ax$.

l) What is the value of $\dfrac{1}{F(D)}[e^{ax}v(x)]$.

Part-II

2. Answer any ***eight*** of the following : 2 × 8

a) Identify the surface $36x^2 + 9y^2 + 4z^2 = 36$ and their intersection with the principal plane.

b) Find the asymptote parallel to co-ordinate axes $x^2y^2 - a^2(x^2 + y^2) = 0$.

c) Find the radius of curvature of $S = a\psi$.

d) Verify Rolle's theorem for $\frac{\sin x}{e^x}$ in $[0, \pi]$.

e) Determine $\lim\left\{\frac{1}{x-2} - \frac{1}{\log(x-1)}\right\}$ as $x \to 2$.

f) Determine $\lim(\cos x)^{1/x^2}$ as $x \to 0$.

g) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$ does n't exist.

h) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$.

i) Solve $x\frac{dy}{dx} = \sqrt{1-y^2}$.

j) Find the general solution of

$y^{iv} - 4y^{III} + 8y^{II} - 8y^{I} + 4y = 0$

Part-III

3. Answer any ***eight*** of the following : 3×8

a) Prove that the radius of curvature of $(-2a, 2a)$ on the curve
$x^2y = a(x^2 + y^2)$ is $-2a$

b) Find the area lying about x-axis and included between $x^2 + y^2 = 2ax$ and parabola $y^2 = ax$.

c) Rectify the curve
$x = a(\theta + \sin\theta)$ $y = a(1 - \cos\theta)$

d) Derive Maclaurin series expansion of cos x.

e) Find $\lim \dfrac{1 + \sin x - \cos x + \log(1 - x)}{x \tan^2 x}$ as $x \to 0$

f) Find the centroid of a right angular cone of base radius r and height h with uniform density 1.

g) Evaluate $\iint_R \left(1 - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}\right) dxdy$ over the first quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

h) Solve $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$

i) Solve $yy' = x^3 + y^2$, $y(2) = -6$.

j) Show that if z^x is a homogeneous function of x and y of degree n, then

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^2 z = n(n-1)z.$$

Part-IV

4. a) The normal at any point P of the central conicoid $ax^2 + by^2 + cz^2 = 1$ meets the three principal planes at G_1, G_2 and G_3.
Show that $PG_1 : PG_2 : PG_3 :: a^{-1} : b^{-1} : c^{-1}$. 7

OR

b) Find the enveloping cylinder of the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and the equation of whose generators are $x = y = z$.

5. a) State and prove Cauchy Mean Value Theorem. Verify the theorem for

$x^2 \in [a, b]$ a > 0, b > 0. 7

OR

b) Obain the Maclaurin's theorem the first four terms of $e^{x \cos x}$ in ascending powers of x. Show

that $\lim_{x \to 0} \frac{e^x - e^{x \cos x}}{x - \sin x} = 3$.

6. a) Find the maximum value of the function

$f(x, y, z) = \frac{5xyz}{x + 2y + 4z}$ such that the condition

xyz = 8. 7

OR

b) If $f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

7. a) Solve $(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x$. 7

OR

b) Solve tha initial value problem

$$\frac{dy}{dx} + y = f(x) \text{ when}$$

$$f(x) = \begin{cases} 2, & 0 \le x < 1, \quad y(0) = 0 \\ 0, & x \ge 1 \end{cases}$$